

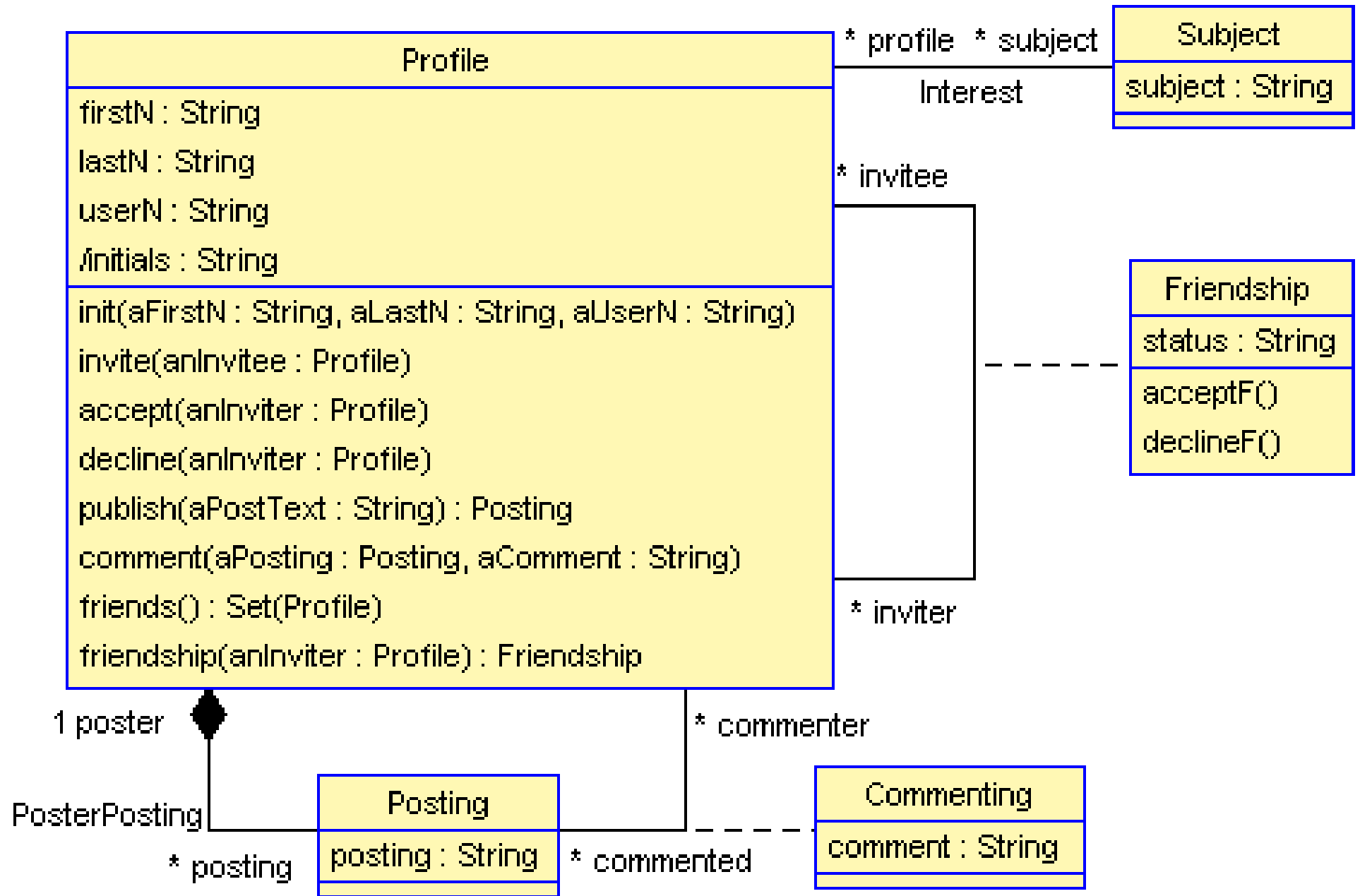
Design of Information Systems

OCL Collection Concepts and Collection Operations

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Collections

- Collections common in modeling and programming languages
- *"A collection (or container) is a **grouping** of some variable number of **data items** (possibly zero) that ... need to be operated upon together in some controlled fashion."* Wikipedia
- Examples: set, list, multi-set (allowing duplicates), stack, ...
- UML collections: Set, Bag, Sequence, OrderedSet, Tuple
- Parametrized with element type(s) and access option (for Tuple)



Example collections in SocialNetwork

```
merkel.inviter: Set(Profile)
```

```
merkel.posting: Set(Posting)
```

```
merkel.posting.commenter: Bag(Profile)
```

```
-- !create merkel,putin,trump:Profile  
Sequence{merkel,putin,trump}: Sequence(Profile)
```

```
OrderedSet{merkel,putin,trump}: OrderedSet(Profile)
```

```
Sequence{merkel,putin,trump,may}.yearE = Sequence{2005,2000,2016,2016}  
-- yearE: year of first election; imaginable for example model
```

```
OrderedSet{2005,2000,2016,2016} = OrderedSet{2005,2000,2016}
```

```
-- Paper::authors:OrderedSet(Author); more precise than  
Sequence(Author)
```

```
Sequence{may,merkel}->collect(p|Tuple{L:p.lastN,I:p.initials}) =  
  Sequence{Tuple{L='May', I='TM'},  
           Tuple{L='Merkel',I='AM'}}:  
Sequence(Tuple(L:String,I:String))
```

Example collections in ConferenceWorld

USE: ConferencePaper.use

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ConferencePaper

- Classes
 - Person
 - Conference
 - Paper
- Associations
 - Program
- Invariants
- Pre-/Postconditions

association Program between
Conference[0..1] role acceptingC
Paper[1..*] role acceptedP
end

Class diagram

```

classDiagram
    class Person
    class Conference {
        SessionChairs : Sequence(Person)
    }
    class Paper {
        Authors : OrderedSet(Person)
    }
    Conference "0..1" -- "1..*" Paper : Program
    
```

Object diagram

```

classDiagram
    class icse : Conference {
        SessionChairs = Sequence{eve,ada,eve}
    }
    class exec4uml : Paper {
        Authors = OrderedSet{bob,ada}
    }
    class checkPrePost : Paper {
        Authors = OrderedSet{bob,cyd}
    }
    class ada : Person
    class cyd : Person
    class eve : Person
    class bob : Person
    class dan : Person
    icse -- exec4uml : Program
    icse -- checkPrePost : Program
    
```

Evaluate OCL expression

Enter OCL expression: icse.acceptedP

Result: Set{checkPrePost,exec4uml} : Set(Paper)

(a)

Evaluate OCL expression

Enter OCL expression: icse.acceptedP.Authors

Result: Bag{ada,bob,bob,cyd} : Bag(Person)

(b)

Evaluate

Browser

Clear

(c)

(d)

Ready.

Collection parameters and collection syntax

- Type kinds with type parameters: Set(T), Bag(T), Sequence(T), OrderedSet(T), Tuple(A1:T1,...,An:Tn); tuple component access Ai
- Abstract type kind (no instances): Collection(T), generalization of Set(T), Bag(T), Sequence(T), OrderedSet(T)
- Parameter actualization in order to build types
- Types (class model level) always written with parentheses ()

```
Set(Posting) , Bag(Profile) ,  
Sequence(Profile) , OrderedSet(Integer) ,  
Tuple(L:String, I:String)
```

- Instantiations (object model level) always written with braces { }

```
Set{merkel, trump} , Bag{trump, putin, trump} ,  
Sequence{merkel, putin, trump} ,  
OrderedSet{2005, 2000, 2016} ,  
Tuple{L='Merkel' , I='AM' }
```

- Tuple access: Tuple{L='Merkel' , I='AM' } . I='AM'

Collection properties (for homogeneous collections)

- Two criteria in order to distinguish between collections:
 (1) Insertion **order** relevance and (2) Insertion **frequency** relevance
- Is the insertion order relevant for distinguishing collections?

$COL \rightarrow \text{including}(E1) \rightarrow \text{including}(E2) = COL \rightarrow \text{including}(E2) \rightarrow \text{including}(E1)$

if required, collection is called **order-blind**, else **order-aware**

- Is the insertion frequency relevant for distinguishing collections?

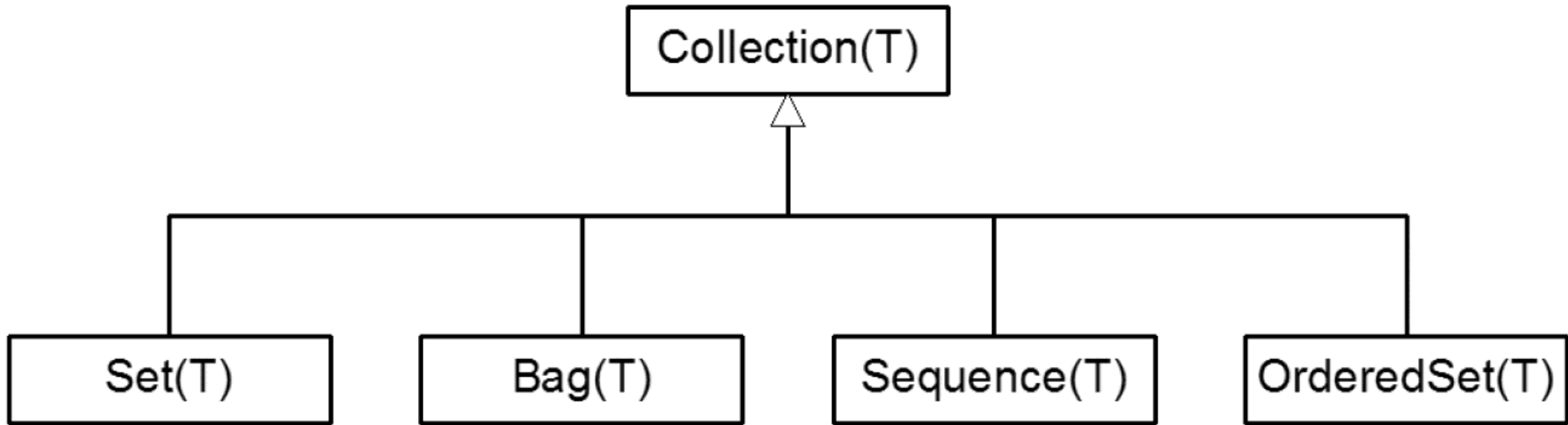
$COL \rightarrow \text{includes}(E) \text{ implies } (COL \rightarrow \text{including}(E) = COL)$

if required, collection is called **frequency-blind**, else **frequency-aware**

-

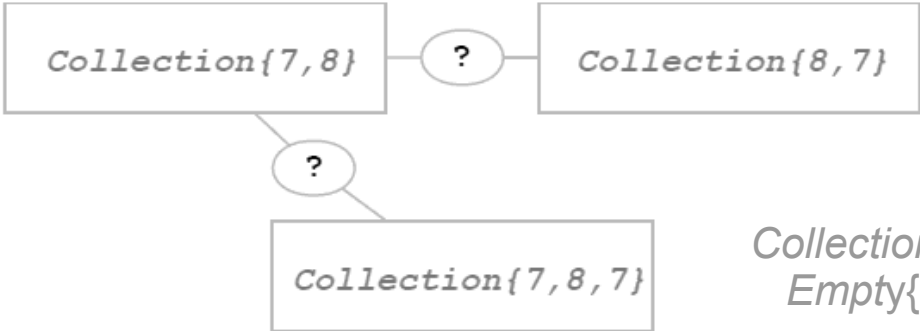
		order	
		blind	aware
frequency	blind	Set (T)	OrderedSet (T)
	aware	Bag (T)	Sequence (T)

Collection type hierarchy and properties

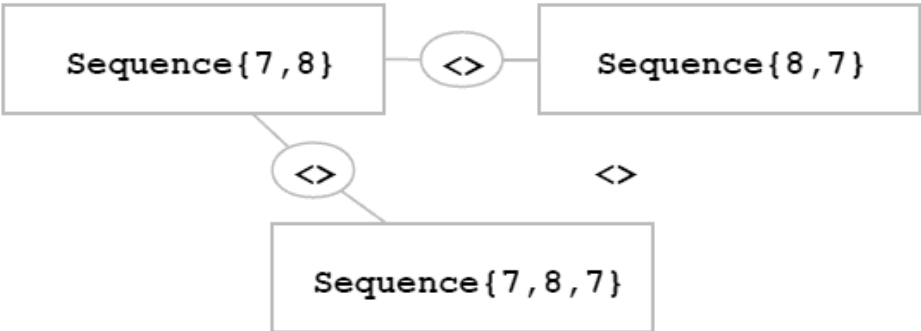
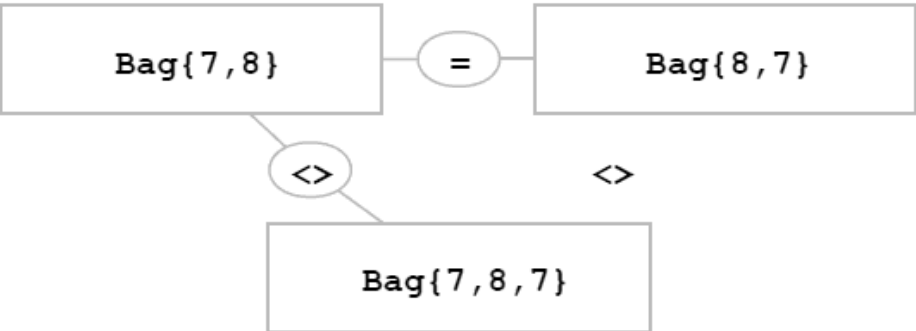
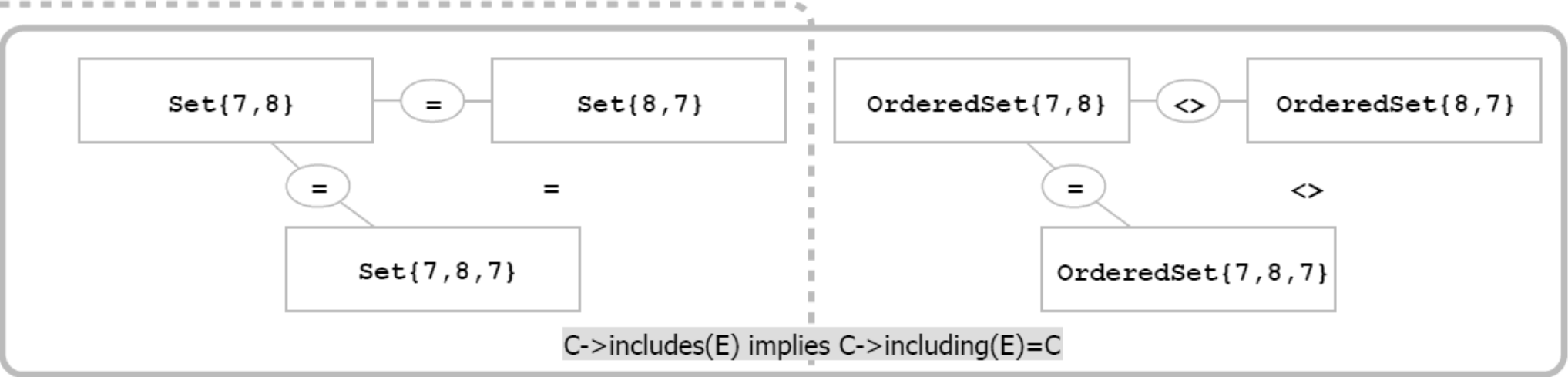


- order-blind and frequency-blind `Set(T)`
- order-blind and frequency-aware `Bag(T)`
- order-aware and frequency-aware `Sequence(T)`
- order-aware and frequency-blind `OrderedSet(T)`
- OCL 1.3 only had `Set(T)`, `Bag(T)`, `Sequence(T)`
- OCL 1.4 added `OrderedSet(T)`
- also used: order-insensible/-sensible, frequency-insensible/-sensible

Collection properties: Insertion order and frequency



$Collection\{x,y\} = Empty\} \rightarrow including(x) \rightarrow including(y)$



$C \rightarrow including(E1) \rightarrow including(E2) = C \rightarrow including(E2) \rightarrow including(E1)$

Collection properties

```
use> !C:=Set{Set{7,8}, Set{8,7},  
             Set{7,8,8}, Set{8,7,7}} 01  
use> ?C 02  
Set{Set{7,8}} : Set(Set(Integer)) 03  
04  
use> !D:=Set{Bag{7,8}, Bag{8,7},  
             Bag{7,8,8}, Bag{8,7,7}} 05  
use> ?D 06  
Set{Bag{7,8}, Bag{7,7,8}, Bag{7,8,8}} : Set(Bag(Integer)) 07  
08  
use> !E:=Set{OrderedSet{7,8}, OrderedSet{8,7},  
             OrderedSet{7,8,8}, OrderedSet{8,7,7}} 09  
use> ?E 10  
Set{OrderedSet{7,8}, OrderedSet{8,7}} : Set(OrderedSet(Integer)) 11  
12  
use> !F:=Set{Sequence{7,8}, Sequence{8,7},  
             Sequence{7,8,8}, Sequence{8,7,7}} 13  
use> ?F 14  
Set{Sequence{7,8}, Sequence{8,7},  
     Sequence{7,8,8}, Sequence{8,7,7}} : Set(Sequence(Integer)) 15  
16  
17  
use> ?Sequence{C->size(), D->size(), E->size(), F->size()} 18  
Sequence{1, 3, 2, 4} : Sequence(Integer) 19
```

Collection operations on all collection kinds

Constructors and `destructors`

- Set{...}, Bag{...}, Sequence{...}, OrderedSet{...}
- Set{L..H}, Bag{L..H}, Sequence{L..H}, OrderedSet{L..H} -- Low High
- including(...), excluding(...)

Basic boolean and integer query operations

- =, <>
- includes(...), excludes(...), includesAll(...), excludesAll(...)
- isEmpty(), notEmpty(), size(), count(...)

Advanced boolean query operations

- forAll(...), exists(...), one(...)
- isUnique(...)

Advanced collection-valued query operations

- select(...), reject(...)
- any(...)
- union(...)
- collect(...), collectNested(...)
- flatten()
- sortBy(...)

Complex query operations: iterate(...), closure(...)

Coercions: asSet(), asBag(), asSequence(), asOrderedSet()

Collection operations on special collection kinds

- `first()`, `last()`, `at(pos)`, `reverse()`
for order-aware, i.e. `Sequence(T)`, `OrderedSet(T)`
- `subSequence(startPos, endPos)` on `Sequence(T)`
- `subOrderedSet(startPos, endPos)` on `OrderedSet(T)`
- `intersection(...)` for order-blind, i.e. `Set(T)`, `Bag(T)`
- `sum()`, `min()`, `max()` on `Collection(Integer)`, `Collection(Real)`
- Few further operations (e.g. `indexOf`): see OCL standard

Not mentioned yet (and to be discussed further down):
collection operations in the context of **generalization**
(e.g. for Chess example, `c:Character` and `c.ocIsTypeOf(Knight)`)

Demonstrating OCL expressions without having objects (Part A)

Constructors and `destructors`

- `Set{7,8}`, `Bag{7,8,8}`, `Sequence{7,8,7}`, `OrderedSet{8,7,7}`
- `Set{}`, `Bag{}`, `Sequence{}`, `OrderedSet{}`
- `Set{7..9}`, `Bag{7..9}`, `Sequence{7..9}`, `OrderedSet{7..9}`
- `Set{}->including(8)->including(7)`, `Bag{8,9,7,8,9}->excluding(9)`

Basic boolean and integer query operations

- `Set{7,8}=Set{8,7,8,7}`, `OrderedSet{7,8}<>OrderedSet{8,7}`
`Set{7,8}<>Bag{7,8}`, `OrderedSet{7,8}<>Sequence{8,7}`
- `Set{7,8}->includes(8)`, `Set{7,8}->excludes(9)`,
`Set{7,8}->includesAll(Set{8,8,7,7})`, `Set{7,8}->excludesAll(Set{6,9})`
- `Set{}->isEmpty()`, `Set{7,8}->notEmpty()`, `Set{8,8,7,7}->size()=2`
`Set{7,8,7}->count(7)`, `Bag{7,8,7}->count(7)`
`Sequence{7,8,7}->count(7)`, `OrderedSet{7,8,7}->count(7)`

Demonstrating OCL expressions without having objects (Part B)

Advanced boolean query operations

- `Set{7..9}->forall(i|i>=0), Bag{7..9}->exists(i|i.mod(2)=0)`
- `Sequence{7..9}->one(i|i.mod(2)=0)`
- `OrderedSet{-9..-8}->including(8)->including(9)->isUnique(i|i*i)=false`

Advanced collection-valued query operations

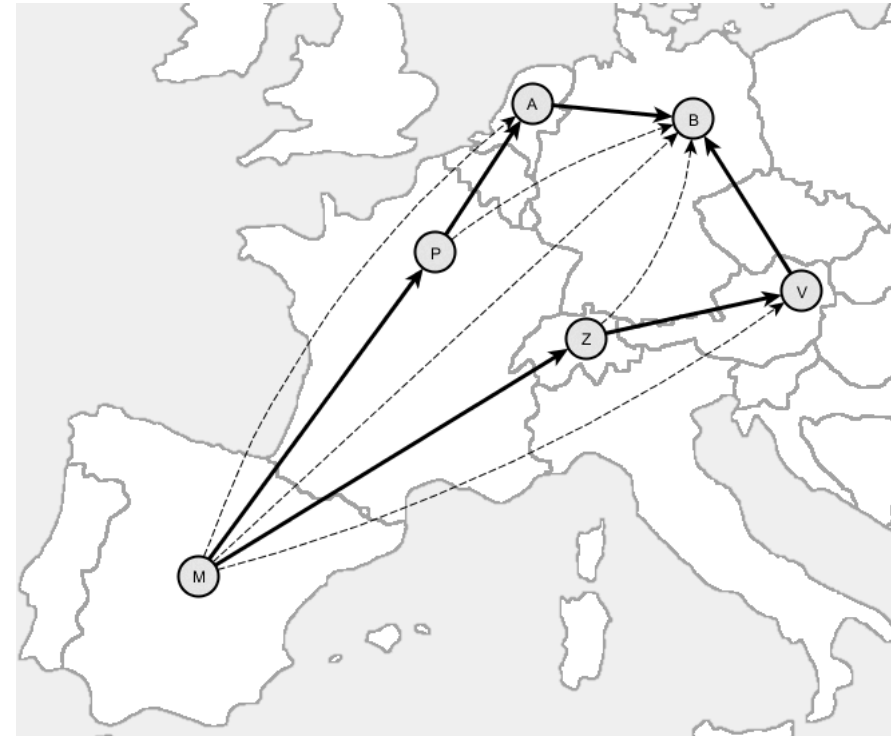
- `Set{21..42}->select(i|i.mod(3)=0 and i.mod(7)=0)`
- `Bag{21..42}->reject(i|i.mod(2)=0 or i.mod(3)=0)`
- `Set{21..42}->any(i|i.mod(2)=1)`
- `Set{7,8,8}->union(Set{9,9,8}), Bag{7,8,8}->union(Bag{9,9,8})`
`Sequence{7,8,8}->union(Sequence{9,9,8})`
`OrderedSet{7,8,8}->union(OrderedSet{9,9,8})`
- `Set{-2..2}->collect(i|i*i), Set{-2..2}->collect(i|Sequence{i,i*i})`
`Set{-2..2}->collectNested(i|Sequence{i,i*i})`
- `Set{-2..2}->collectNested(i|Sequence{i,i*i})->flatten()`
- `Set{-6,-5,-4,7,8,9}->sortedBy(i|i*i)`

Demonstrating OCL expressions without having objects (Part C)

Complex query operations

- `Set{-2..2}->iterate(i:Integer;r:Set(Sequence(OclAny))=Set{}|r->including(Sequence{i,i*i,if i.mod(2)=0 then 'E' else 'O' endif}))`
- Capitals: M[adrid], P[aris], A[msterdam], B[erlin], Z[urich], V[ienna]
let TupleSet=
Set{Tuple{s:'M',t:'P'},Tuple{s:'P',t:'A'},Tuple{s:'A',t:'B'},
Tuple{s:'M',t:'Z'},Tuple{s:'Z',t:'V'},Tuple{s:'V',t:'B'}} in
TupleSet->closure(T1|
TupleSet->select(T2|T1.t=T2.s)->
collect(T2|Tuple{s:T1.s,t:T2.t}))

```
select =
+-----+
|               |
Tuple{T1.s,T1.t}  Tuple{T2.s,T2.t}
|               |
+-----+
collect
constructs new, transitive tuple
```



Demonstrating OCL expressions without having objects (Part D)

Coercions

- `Sequence{8,7,8}->asSet()=Set{8,7}`
- `OrderedSet{8,7,8}->asBag()=Bag{8,7}`
- `Set{7,8}->asSequence()=Sequence{8,7}`
or `Set{7,8}->asSequence()=Sequence{7,8}`
- `Bag{8,8,7,7}->asOrderedSet()=OrderedSet{7,8}`
or `Bag{8,8,7,7}->asOrderedSet()=OrderedSet{8,7}`
- `Set{-2..2}->collect(i|i*i)->asSet()`

Collection operation iterate for iterations

- COLEXPR->iterate (ELEMVAR:ELEMTYPE; RESVAR:RESTYPE=INITEXPR | ITEREXPR)
 - COLEXPR, INITEXPR, ITEREXPR: OCL expression
ELEMVAR, RESVAR: OCL variables
ELEMTYPE, RESTYPE: OCL types
ITEREXPR may use ELEMVAR, RESVAR; ITEREXPR not forced to do so
- type* (COLEXPR) in
{Set (ELEMTYPE), Bag (ELEMTYPE), Sequence (ELEMTYPE), OrderedSet (ELEMTYPE)}
type (INITEXPR) = *type* (ITEREXPR) = RESTYPE
- Also allowed: COLEXPR->iterate (ELEMVAR; RESVAR:RESTYPE=INITEXPR | ITEREXPR)
i.e., ':ELEMTYPE' is optional
 - Collection operations can be expressed with iterate
 - Example

```
ibm.worker->exists (p:Person | p.fName='Bob')
```

```
ibm.worker->iterate (p:Person; bobEx:Boolean=false | bobEx or p.fName='Bob')
```

```
COLEXPR   ibm.worker
```

```
ELEMVAR   p
```

```
ELEMTYPE  Person
```

```
RESVAR    bobEx
```

```
RESTYPE   Boolean
```

```
INITEXPR  false
```

```
ibm.worker = Set{ada,bob} ->
```

```
ITEREXPR  bobEx or p.fName='Bob' false or ada.fName='Bob' or bob.fName='Bob'
```

- iterate Evaluation in Java-like Pseudo Code

COLEXPR->iterate(ELEMVAR:ELEMTYPE; RESVAR:RESTYPE=INITEXPR | ITEREXPR)

```
RESTYPE iterate() {
    ELEMTYPE ELEMVAR;
    RESTYPE RESVAR = INITEXPR;
    for (Iterator i = COLEXPR.iterator(); i.hasNext();) {
        ELEMVAR = (ELEMTYPE)i.next();
        RESVAR = ITEREXPR;
    };
    return RESVAR;
}
```

- Expressing other collection operation with iterate; given COL:Set(T)

COL->select(e | p(e)) ==>

COL->iterate(e; r:Set(T)=Set{} | if p(e) then r->including(e) else r endif)

COL->collect(e | t(e)) ==> COL->iterate(...)

COL->forAll(e | p(e)) ==> COL->iterate(e; r:Boolean=true | r and p(e))

COL->iterate(e; r:Boolean=true | false) <== COL->ColOpXYZ()

COL->size() ==> COL->iterate(e; sz:Integer=0 | sz+1)

...

Excursion: Transitive closure of a relation

https://en.wikipedia.org/wiki/Transitive_closure



Transitive closure

In [mathematics](#), the **transitive closure** of a [binary relation](#) R on a [set](#) X is the smallest relation on X that contains R and is [transitive](#).

relation transitive: aRb and bRc implies aRc

For example, if X is a set of airports and xRy means "there is a direct flight from airport x to airport y " (for x and y in X), then the transitive closure of R on X is the relation R^+ such that xR^+y means "it is possible to fly from x to y in one or more flights". Informally, the *transitive closure* gives you the set of all places you can get to from any starting place.

Existence and description [\[edit \]](#)

For any relation R , the transitive closure of R always exists. To see this, note that the [intersection](#) of any [family](#) of transitive relations is again transitive. Furthermore, [there exists](#) at least one transitive relation containing R , namely the trivial one: $X \times X$. The transitive closure of R is then given by the intersection of all transitive relations containing R .

For finite sets, we can construct the transitive closure step by step, starting from R and adding transitive edges. This gives the intuition for a general construction. For any set X , we can prove that transitive closure is given by the following expression

$$R^+ = \bigcup_{i=1}^{\infty} R^i.$$

where R^i is the i -th power of R , defined inductively by

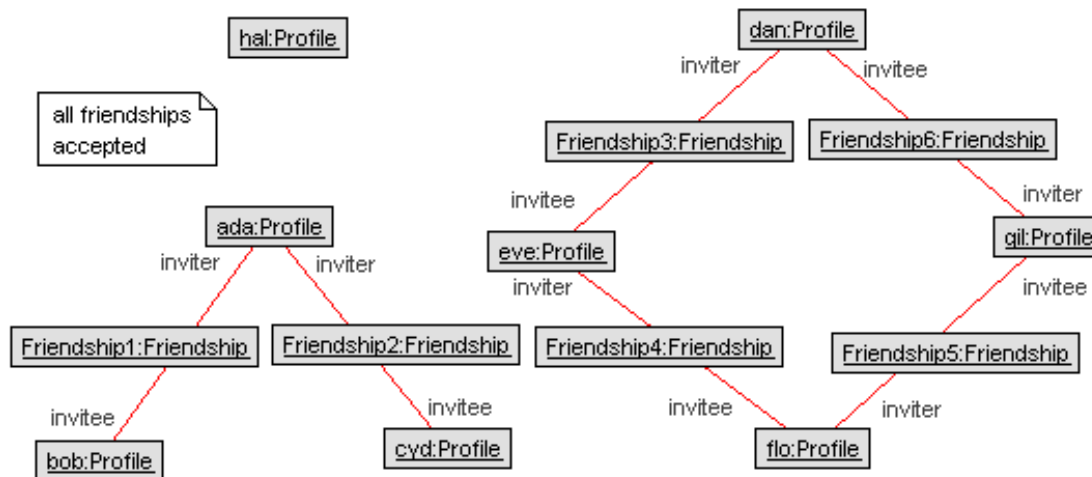
$$R^1 = R$$

and, for $i > 0$,

$$R^{i+1} = R \circ R^i \quad \text{where } \circ \text{ denotes } \text{composition of relations}.$$

Collection operation closure for transitive closure and cycles

- COL : Collection(C) ; C::CLOSURE_TERM:Collection(C)
CLOSURE_TERM: role, attr, query operation or collection operation on them
- COL->closure(CLOSURE_TERM)
COL->closure(ELEMVAR | CLOSURE_TERM)
COL->closure(ELEMVAR:ELEMTYPE | CLOSURE_TERM)
- Given C::term:Set(C) and c:C :
c.term->closure(term) = transitive closure; c included if reachable by term
Set{c}->closure(term) = reflexive, transitive closure; c always included



Evaluate OCL expression

Enter OCL expression:

Result:

Evaluate OCL expression

Enter OCL expression:

Result:

Evaluate OCL expression

Enter OCL expression:

Result:

Evaluate OCL expression

Enter OCL expression:

Result:

Evaluate OCL expression

Enter OCL expression:

Result:

Evaluate OCL expression

Enter OCL expression:

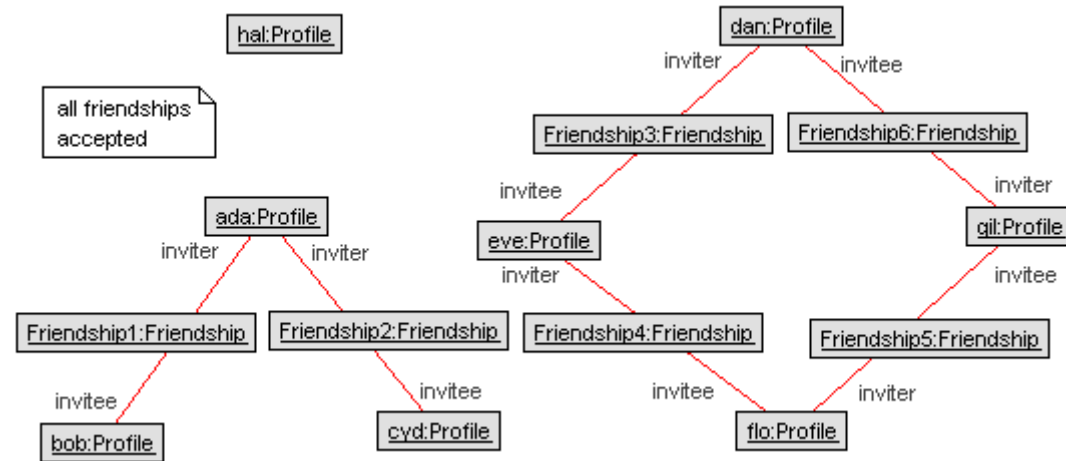
Result:

Collection operation closure - Further examples

```
use> ?ada.friends()  
Set{bob,cyd} : Set(Profile)  
use> ?bob.friends()  
Set{ada} : Set(Profile)  
use> ?cyd.friends()  
Set{ada} : Set(Profile)
```

```
use> ?ada.friends()->closure(friends())  
Set{ada,bob,cyd} : Set(Profile)  
use> ?bob.friends()->closure(friends())  
Set{ada,bob,cyd} : Set(Profile)  
use> ?cyd.friends()->closure(friends())  
Set{ada,bob,cyd} : Set(Profile)
```

```
use> ?ada.inviter->union(ada.invitee)  
Set{bob,cyd} : Set(Profile)  
use> ?bob.inviter->union(bob.invitee)  
Set{ada} : Set(Profile)  
use> ?cyd.inviter->union(cyd.invitee)  
Set{ada} : Set(Profile)
```



```
use> ?ada.inviter->union(ada.invitee)->closure(inviter->union(invitee))  
Set{ada,bob,cyd} : Set(Profile)  
use> ?bob.inviter->union(bob.invitee)->closure(inviter->union(invitee))  
Set{ada,bob,cyd} : Set(Profile)  
use> ?cyd.inviter->union(cyd.invitee)->closure(inviter->union(invitee))  
Set{ada,bob,cyd} : Set(Profile)
```

```
use> ?dan.inviter  
Set{gil} : Set(Profile)  
use> ?dan.inviter->closure(inviter)  
Set{dan,eve,flo,gil} : Set(Profile)
```

Collection operation closure - Classical example: Acyclic parenthood

USE: person.use

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OCL

Parenthood

- Classes
 - Person
- Associations
 - Parenthood
- Invariants
 - Person::acyclicParenthood
- Pre-/Postconditions
- Query Operations

USE brings original, short expression ... closure(parent) into a form with an explicit variable like ... closure(p [:Person] | p.parent)

Class diagram

```

classDiagram
    class Person
    class Parenthood
    Person "0..2" -- "1" Parenthood
    Person "*" -- "*" Parenthood : child
    
```

context p:Person inv acyclicParenthood: p.parent->closure(parent)->excludes(p)

context p : Person inv acyclicParenthood:
p.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(p)

LEFT OBJECT DIAGRAM

```

bob.child->closure(child)
Set(dan,eve) : Set(Person)

bob.parent->closure(parent)
Set(ada) : Set(Person)
    
```

RIGHT OBJECT DIAGRAM

```

bob.child->closure(child)
Set(ada,bob,cyd,dan,eve) : Set(Person)

bob.parent->closure(parent)
Set(ada,bob,eve) : Set(Person)
    
```

Object diagram

```

graph TD
    ada[ada:Person] -- parent --> bob[bob:Person]
    ada -- parent --> cyd[cyd:Person]
    bob -- child --> dan[dan:Person]
    bob -- child --> eve[eve:Person]
    cyd -- child --> dan
    cyd -- child --> eve
    
```

Object diagram

```

graph TD
    ada[ada:Person] -- parent --> bob[bob:Person]
    ada -- parent --> cyd[cyd:Person]
    bob -- child --> dan[dan:Person]
    bob -- child --> eve[eve:Person]
    cyd -- child --> dan
    cyd -- child --> eve
    dan -- parent --> cyd
    
```

Class invariants

Invariant	Satisfied
Person::acyclicParenthood	true
Cnstrs. OK. (13ms) 100%	

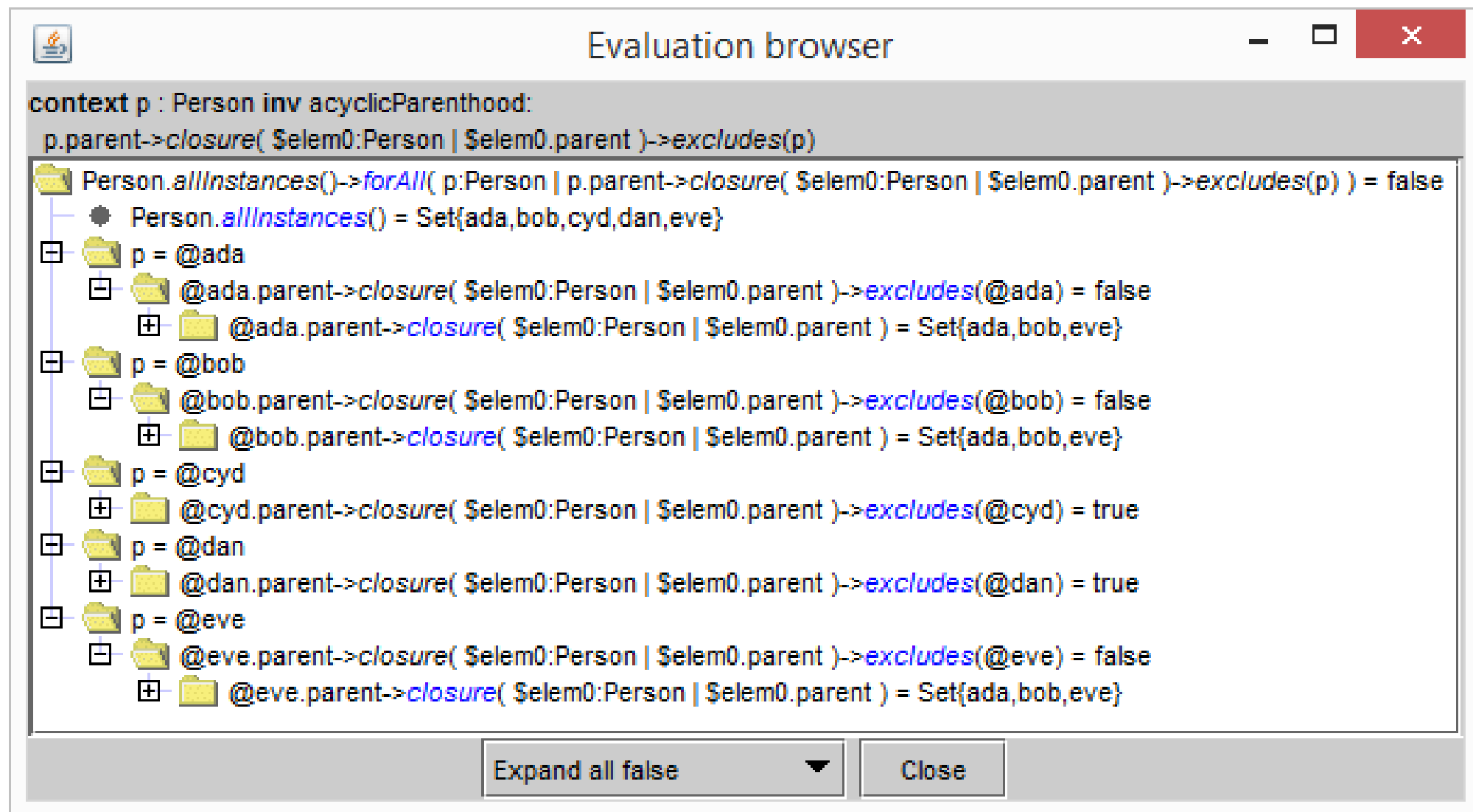
Class invariants

Invariant	Satisfied
Person::acyclicParenthood	false
1 cnstr. failed. Inherent cnstrs. OK. (4ms) 100%	

Ready.

Collection operation closure - Analysis with USE Evaluation Browser

- Double-clicking the failing invariant opens the Evaluation Browser Window
- Window can be tuned through context menu and bottom selection box to explore which objects contribute to invariant failure



The screenshot shows a window titled "Evaluation browser" with a standard Windows-style title bar (minimize, maximize, close buttons). The main content area displays the following text:

```
context p : Person inv acyclicParenthood:  
p.parent->closure( $elem0:Person | $elem0.parent )->excludes(p)
```

Below this, a tree view shows the evaluation results for the invariant. The root node is "Person.allInstances()->forall(p:Person | p.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(p)) = false". The tree is expanded to show the following nodes:

- Person.allInstances() = Set{ada,bob,cyd,dan,eve}
- p = @ada
 - @ada.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(@ada) = false
 - @ada.parent->closure(\$elem0:Person | \$elem0.parent) = Set{ada,bob,eve}
- p = @bob
 - @bob.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(@bob) = false
 - @bob.parent->closure(\$elem0:Person | \$elem0.parent) = Set{ada,bob,eve}
- p = @cyd
 - @cyd.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(@cyd) = true
- p = @dan
 - @dan.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(@dan) = true
- p = @eve
 - @eve.parent->closure(\$elem0:Person | \$elem0.parent)->excludes(@eve) = false
 - @eve.parent->closure(\$elem0:Person | \$elem0.parent) = Set{ada,bob,eve}

At the bottom of the window, there are two buttons: "Expand all false" and "Close".

Thanks for your attention!